

Analytical Methodology for Predicting Widespread Fatigue Damage Onset in Fuselage Structure

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A comprehensive analytical methodology has been developed for predicting the onset of widespread fatigue damage (WFD) in fuselage structure. The determination of the number of flights and operational hours of aircraft service life that are related to the onset of WFD includes analyses for crack initiation, fatigue crack growth, and residual strength. Therefore, the computational capability required to predict analytically the onset of WFD must be able to represent a wide range of crack sizes, from the material (microscale) level to the global (structural-scale) level. The results of carefully conducted teardown examinations of aircraft components indicate that fatigue crack behavior can be represented conveniently by the following three analysis scales: 1) small three-dimensional cracks at the microscale level, 2) through-the-thickness two-dimensional cracks at the local structural level, and 3) long cracks at the global structural level. The computational requirements for each of these three analysis scales are described in this paper.

Introduction

THE ability to predict analytically the onset of widespread fatigue damage (WFD) in fuselage structures requires methodologies that predict fatigue crack initiation, crack growth, and residual strength. Mechanics-based analysis methodologies are highly desirable because differences in aircraft service histories can be addressed explicitly and rigorously by analyzing different types of aircraft and specific aircraft within a given type. Each aircraft manufacturer has developed mature in-house durability and damage-tolerance design and analysis methodologies that are based on their product development history. To enhance these existing successful methodologies, NASA has adopted the concept of developing an analytical tool box, which includes a number of advanced structural analysis computer codes that, taken together, represent the comprehensive fracture mechanics capability required to predict the onset of WFD. These structural analysis tools have complementary and specialized capabilities, ranging from a nonlinear finite element-based stress-analysis code for two- and three-dimensional built-up structures with cracks, to a fatigue and fracture analysis code that uses stress-intensity factors and material property data found in look-up tables or from equations. The development of these advanced structural analysis methodologies has been guided by the physical evidence of the fatigue process assembled from detailed teardown examinations of actual aircraft structure. In addition, critical experiments are being conducted to verify the predictive capability of these codes and to provide the basis for any further methodology refinements that may be required. This paper presents the analytical methodology developed by the authors for predicting the onset of WFD. After a discussion of the results of a thorough teardown fractographic inspection of a five-bay fuselage lap-splice joint containing WFD, the analytical fracture

mechanics requirements to predict crack initiation, fatigue crack growth, and residual strength are presented. For each of these three fracture mechanics scales, example calculations will be compared to the results of experimental verification tests.

The analytical methodology developed by the authors is deterministic and is based on fracture mechanics principles. Advanced elastic–plastic fatigue and fracture criteria have been experimentally verified and implemented into conventional finite element structural analysis codes. There are alternative stress analysis methods, fracture criteria, and fatigue crack initiation and growth criteria currently under development by other research teams. These alternative approaches are briefly reviewed in the following paragraphs.

An engineering method was developed for analyzing a cracked stiffened structure.¹ This method is based on displacement compatibility and is an economic tool for conducting parametric studies to design damage-tolerant aircraft structure. This approach was used to accurately calculate the residual strength of a wide variety of test panels with simulated multiple-site damage (MSD).^{2–4} A simple strength of materials type of analysis method was developed to account for the secondary bending in riveted lap-splice joints, and this method was used to accurately predict the fatigue life of riveted flat panels.⁵ Conventional finite element analyses have also been used to successfully predict the failure of panels with MSD. For example, the finite element method and the J-integral fracture parameter were used so that simple fracture mechanics (linear, elastic) could be used to predict residual strength.⁶ The finite element method was used to analyze the fatigue behavior of riveted panels, and showed the importance of secondary bending stresses and fastener clamp-up stresses on the fatigue predictions.⁷ A very innovative finite element alternating method to calculate stress-intensity factors was developed that is more computationally efficient than conventional finite element methods.⁸ The elastic finite element alternating method⁸ and the elastic–plastic finite element alternating method⁹ are analysis procedures for cracks in a finite body that iterate between the analytical solution for an embedded crack in the infinite domain and the finite element solution for the un-cracked body. Specialized boundary element methods also were developed for calculating the stress-intensity factors for MSD crack configurations.^{10,11} These newer approaches exploit the modeling simplicity of the conventional boundary-element analysis, but use specialized fracture mechanics mathematical

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expressions for the crack face elements, so that very accurate finite body stress-intensity-factor solutions are computed. A number of probabilistic methods are also being developed. A code, PROF, was developed¹² that is used extensively by the U.S. Air Force for conducting risk assessments of fracture-critical structural components. PROF calculates a time history of fracture probability as a function of flight hours by accounting for crack growth, crack detection and repair at inspections, and variability in fracture toughness. A probabilistic-based computer code, INSIM, was developed¹³ that is used by Israel Aircraft Industries, Inc., to simulate the entire aircraft fatigue environment and determine inspection intervals. A probabilistic model was developed for analyzing the effects of MSD on fatigue life.¹⁴ Probabilistic models also were developed for predicting the fatigue lives of cracks that nucleate from constituent particles.^{15,16}

Although the authors are advocating the critical crack-tip opening angle (CTOA) fracture criterion, other elastic-plastic criteria are also under investigation. The K R-curve was successfully used to predict the residual strength of a variety of very wide flat panels with a lead crack and MSD crack configurations.¹⁷ The R-curve was determined from the test of a center-cracked panel with the same lead crack and panel width as the MSD panels. [The test program, conducted by the National Institute of Standards and Technology (NIST)¹⁷ under funding by the Federal Aviation Administration, will be discussed in detail subsequently in the present paper.] A simple MSD linkup criterion was proposed that is based on linear elastic fracture mechanics and Irwin's estimate of the crack-tip plastic zone.¹⁸ It was postulated that linkup will occur when the plastic zones from two adjacent cracks touch.¹⁸ This criterion has been used to accurately predict the linkup loads for flat panels with various panel widths and MSD crack configurations.²⁴ Also, the criterion was modified by using the Dugdale plastic zone model, accurately predicting the linkup and fracture of the very wide MSD panels tested by NIST.³ Use of the J elastic-plastic fracture parameter was advocated because the criterion can be used with simple linear, elastic stress-analysis methods that are in common use throughout the aerospace industry.⁶ It was postulated that the T* integral be used as an elastic-plastic fracture criterion; the original J integral was modified to account for stable crack growth and the subsequent unloading of the elements left in the wake of the advancing crack.¹⁹ The criterion was successfully used to accurately predict the linkup of the MSD cracks and fracture of the very wide panels tested by NIST.²⁰

The authors have developed a deterministic, fracture mechanics method based on the small-crack theory to predict the growth of small cracks that initiate at material microstructural defects in high local stress fields. A similar probabilistic-based approach is being developed.^{15,16} For cracks that do not initiate at the maximum local stress, such as those that initiate from fretting, alternative methods need to be developed. A mechanics-based model was developed to predict fretting fatigue crack nucleation and growth.²¹ Crack nucleation models also were developed for fretting fatigue and corrosion fatigue.²² Finally, the equivalent initial flaw size (EIFS) approach has been extensively advocated as a reasonable engineering approach for using fracture mechanics to predict fatigue life.²³⁻²⁶ The definitive work of Rudd²³ led to the implementation of the EIFS approach into the *U.S. Air Force Durability Design Handbook*.²⁴ More recently, the suitability of the method for predicting service life to the onset of WFD was investigated.²⁵ Because the probabilistic approach to predicting fatigue crack growth and life requires an initial crack population distribution function, the EIFS approach is frequently adopted by the developers of probabilistic methods.²⁶

Fractography of WFD in a Structural Fatigue Test Article

Valid analytical methodology to predict the onset of WFD in fuselage structure must be based on actual observations of

the physical behavior of crack initiation, crack growth, and fracture. The methodology presented herein is based largely on the results of teardown fractographic examinations of aircraft fuselage components. A large section of a fuselage containing a longitudinal lap-splice joint extending for five bays was provided to NASA by an aircraft manufacturer after conducting a full-scale fatigue test.²⁷ A photograph of the panel along with a schematic are shown in Fig. 1. The fatigue test was terminated after reaching the number of fuselage pressurization cycles that equaled approximately three times the original economic design life goal of the aircraft established by the manufacturer. This section of the fuselage was selected because visual inspections made during the test had detected the growth of fatigue cracks extending from adjacent rivets, which eventually linked up to form a long crack that extended completely across the bay. Further visual examinations of this section of the fuselage after completing the full-scale fatigue test suggested that this section contained WFD. All rivet holes in each of the five bays of the panel were microscopically examined for fatigue cracks. The results of this examination form the physical basis for the analytical methodology developed by NASA to predict the onset of WFD.

There were three principal objectives of the fractographic examination of the fuselage panel. The first objective was to characterize WFD in a fuselage splice joint by assembling a database on the initiation and growth of fatigue cracks from rivets, including identifying the initiation mechanisms. The second objective was to provide a basis for comparing the crack growth behavior simulated in laboratory test specimens to the real behavior of an actual aircraft component. The third objective was to serve as a benchmark to verify the predictive capability of the fatigue crack-growth portions of the WFD analytical methodology. This latter objective was achievable because the loading history of the full-scale fatigue test article was fully documented. Also, periodic reductions in the maximum value of the pressure loading cycle (referred to as an underload) were used during the fatigue test to establish marker bands on the fatigue crack surfaces.

Achieving the three objectives resulted in the development of a very large database. This database included 1) maps of cracks as a function of rivet locations in five consecutive bays; 2) documentation of crack-growth shapes and dimensions; 3) identification of the crack initiation location and the initiating mechanisms, such as high local stress, fretting, and manufacturing defects; 4) analysis of fatigue marker bands; 5) correlation between cycles and crack-growth behavior; and 6) indications of out-of-plane displacements and mixed-mode fracture behavior. In addition to the extensive database assembled from the teardown fractographic examinations of the panel, sections of other retired aircraft and full-scale fatigue articles also were examined to ensure that the analytical methodology under development is sufficiently comprehensive to represent all fuselage assembly practices and design details.

While the fractography database is exclusively for a longitudinal-lap splice joint with counterbore rivets, several general

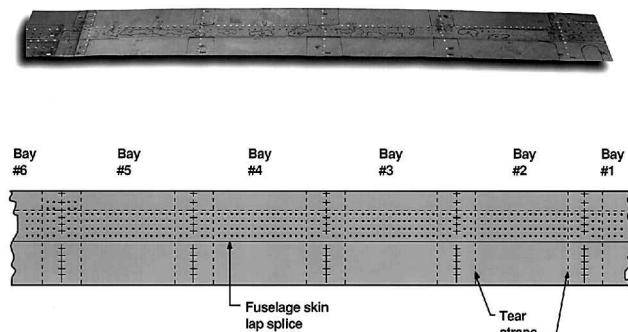


Fig. 1 Fractography of WFD in structural fatigue test article.

conclusions are obvious from the database. First, fatigue cracks were present at virtually every rivet hole in the top row of rivets. The cracks ranged in size from about 50 μm to several centimeters. Cracks were initiated at regions of high stress concentration and by fretting along the mating surfaces. The cracking behavior in each bay was similar, and the results of the fatigue marker-band analysis were relatively independent of rivet hole location. An example of small cracks found in the panel is shown in Fig. 2. A small crack initiating from high local stresses within the rivet countersunk hole is shown in Fig. 2a. The accompanying schematic shows the location of the crack in the outer skin of the lap-splice joint. An example of a small crack initiating from fretting is shown in Fig. 2b, along with a schematic showing the interface where the fretting occurred. Examples of long cracks found at rivet holes are shown in Fig. 3. Figures 3a–3c show a crack that initiated from high local stresses, and Figs. 3d–3f show a crack in a different rivet hole that initiated by fretting. The higher magnifications of the cracks shown in Figs. 3b and 3f help to identify the location where the crack initiated and the initiation mechanism. In Fig. 3c, it is seen that the crack has grown completely through the thickness of the outer skin and has extended a considerable distance beyond the head of the rivet. Likewise, the crack shown in Fig. 3e has also extended a considerable distance from the head of the rivet, but has not broken through the outer surface of the skin. In both cases, the crack front is curved, and indicates the existence of significant bending stresses across the lap-splice joint. A large crack that has been formed by the linkup of the small fatigue cracks that developed at adjacent rivet holes is shown in Fig. 4. As can be seen in the photograph, the crack extended into the tear strap region, changed crack-growth directions, and grew into a rivet hole in the tear strap. The surfaces of the individual fatigue cracks between the rivets were clearly identifiable in the fractographic examination of the long crack surface. Close examination revealed several cracks that initiated from high local stresses, and other cracks that initiated from fretting.

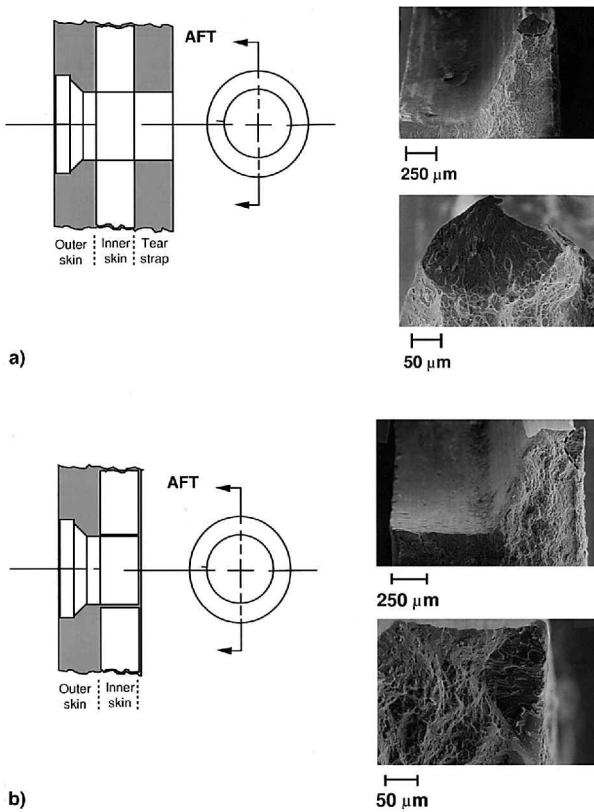


Fig. 2 Small fatigue cracks at rivet holes initiated by a) local high stresses and b) fretting.

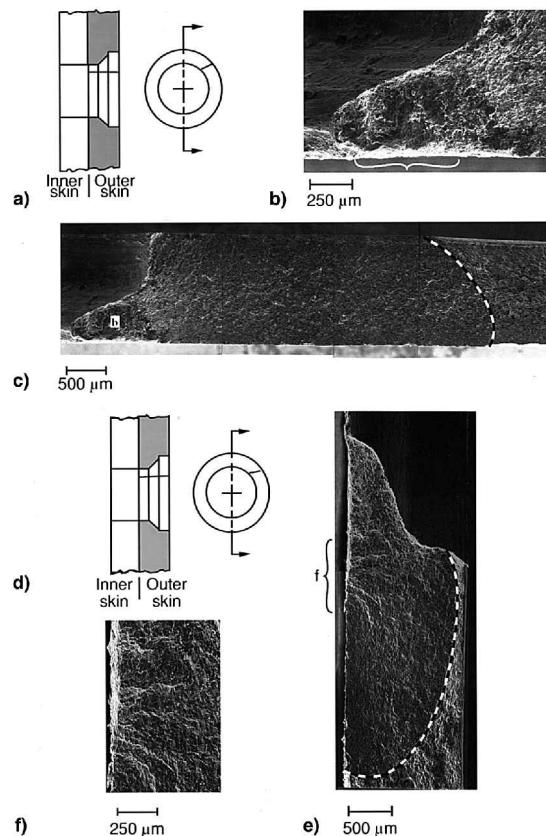


Fig. 3 Two examples of long cracks at rivets.

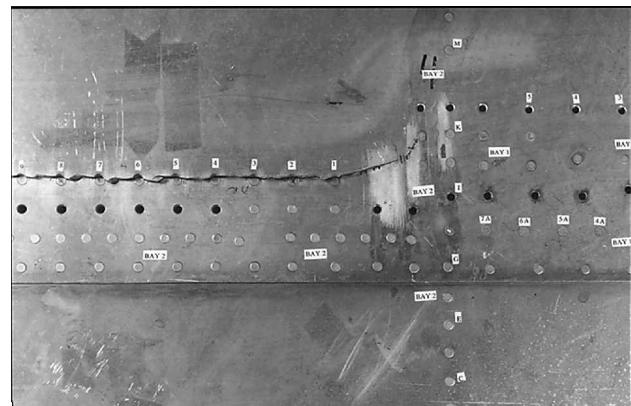


Fig. 4 Fatigue crack in a fuselage splice joint of a transport aircraft.

However, the lengths of all of the fatigue cracks at linkup were approximately the same. This observation suggests that the long-crack behavior is somewhat independent of the initiating mechanism.

To better understand the fatigue crack-growth rates exhibited by the full-scale fatigue test article, a series of underloads was included in the test procedure every 10,000 fuselage pressurization cycles to provide fatigue crack face markers. The photomicrograph shown in Fig. 5 illustrates a typical set of marker bands. This particular set of marker bands was produced by a sequence of several underloads followed by the normal test pressure load. Each 10,000 pressure cycles used a different sequence, so that the precise loading history of each set of marker bands was known. The analysis of these marker bands provided the fatigue crack front location and shape and growth rate between the 10,000 pressure cycle intervals. The experimentally determined crack-growth rates are shown in Fig. 6 as a function of crack length for a number of fatigue cracks found

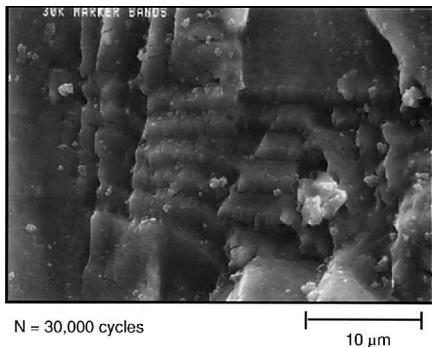


Fig. 5 Marker bands on a fatigue crack face.

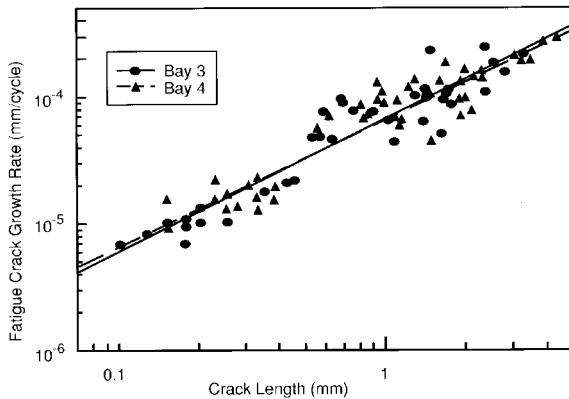


Fig. 6 Crack growth data obtained from the fatigue marker bands.

at rivets in bays 3 and 4. A linear regression fit to the data is also shown to provide a frame of reference for interpreting the experimental data. The analysis of the marker-band data began after the first set of marker bands appeared on the fracture surface. Therefore, the data in Fig. 6 can only be rigorously interpreted for crack growth. At best, the marker-band procedure can only indirectly be interpreted relative to the number of pressure cycles before crack initiation. The data plotted in Fig. 6 include cracks that initiated as a result of high local stresses as well as from fretting. Although the first set of marker bands to appear on the various fatigue crack surfaces varied between 10,000 and 30,000 cycles, there was no discernible difference in the crack-growth rates after initiation, regardless of the initiating mechanism. These data clearly illustrate that the fatigue crack growth is well behaved, and that there is no appreciable difference between the two adjacent bays. (A thorough analysis of the marker-band data may be found in Ref. 28.) The quantitative data obtained from the marker-band analyses and the observations discussed in the preceding paragraphs strongly suggest that the fatigue behavior of the long cracks is deterministic and predictable. These results also suggest that an EIFS approach may be useful in predicting the fatigue life of riveted joints, with a different EIFS being defined for each unique crack initiation mechanism.

Analytical Framework for the Methodology to Predict the Onset of WFD

The fractographic examinations described in the preceding text suggest that the computational capability required to predict analytically the onset of WFD must be able to represent a wide range of crack sizes, from the material (microscale) level to the global (structural-scale) level. These studies indicate that the fatigue crack behavior in aircraft structure can be represented conveniently by the following three analysis scales: 1) small three-dimensional crack geometries at the mi-

croscale level (crack initiation), 2) through-the-thickness two-dimensional crack geometries at the local structural level (fatigue crack growth), and 3) long cracks at the global structural level (residual strength). The computational requirements for each of these three analysis scales are described in the following text.

Scale 1: Crack Initiation

The first analysis scale and corresponding computational capability represents the fracture mechanics of small cracks that exhibit three-dimensional crack-growth behavior. The existence and growth of these small cracks do not affect the global structural deformation states or internal load distributions. Examples of these cracks are surface and corner cracks that initiate at the edges of plates or at holes.

Criterion for Crack Initiation Based on Small-Crack Behavior

Small fatigue cracks in some materials grow faster and at lower stress-intensity-factor levels than is predicted from large-crack data, which exhibits an apparent threshold for crack growth.²⁹ The initiation and growth of these small cracks are affected by metallurgical features such as inclusion particles and grain-boundary interactions.³⁰ Typical large-crack results are obtained from tests with cracks greater than about 2 mm in length. The large-crack threshold is usually obtained from load-reduction tests. Some tests and analyses have shown that the development of the threshold may be caused by an increase in crack-closure behavior as the load is reduced. Small cracks that initiate at inclusion particles, voids, or weak grains do not have any prior plastic deformation to develop crack closure. If a small crack is fully open, then the stress-intensity-factor range is fully effective, and the growth rate for the small crack is faster than the rate exhibited by the large-crack data, and at a lower stress-intensity-factor range.

The concept of using crack closure to explain crack-growth acceleration and retardation was pioneered at NASA Langley Research Center almost two decades ago.³¹ The closure concept is based on the postulate that the wake of plastically deformed material behind an advancing crack front may prevent the crack from being fully open during the complete loading cycle. Therefore, only part of the load cycle is effective in growing the crack. The crack closure concept has also been successfully used to explain the small-crack phenomenon exhibited by many aluminum alloys. The successful coupling of the closure methodology with the small-crack-growth rate database has resulted in a total-life prediction methodology, which treats initiation by predicting the growth of micron-size cracks initiating at inclusion particles in the subgrain boundary microstructure.³²

Computational Methodology for Predicting Crack Initiation in Riveted Structure

Stress-intensity-factor solutions are typically obtained from computational procedures such as the finite element analysis method. The ZIP3D computer code has been developed to model three-dimensional crack configurations and to calculate the corresponding stress-intensity factors.³³ This finite element analysis code uses an eight-node element, and can be used to analyze stationary and growing cracks under cyclic elastic-plastic conditions, including the effects of crack closure. The FRANC3D code also has solid modeling capabilities for three-dimensional geometries based on the boundary element method.³⁴ For those crack configurations and general loading conditions that may occur for various structural components, weight-function solutions are being developed from the numerical results of parametric studies. These weight-function equations are particularly useful because the stress-intensity-factor solutions can be obtained from a stress analysis of the uncracked structure. Stress-intensity-factor solutions are currently being generated for cracks that initiate at countersunk rivet holes. Loading conditions include interference-fit stresses,

clamp-up stresses, and loads transferred through a rivet. These stress-intensity-factor solutions may then be used as input data for the FASTRAN II code to predict fatigue crack growth.³⁵ The FASTRAN II code is based on the mechanics of plasticity-induced crack closure. The effects of prior loading history on fatigue behavior, such as crack-growth retardation and acceleration, are computed on a cycle-by-cycle basis. The code will predict the growth of cracks exhibiting the small-crack effect, as well as of two- and three-dimensional cracks exhibiting the classical Paris law crack-growth behavior. The code has been shown to be particularly effective for predicting fatigue crack-growth behavior in structures subjected to aircraft spectrum loads. The ZIP3D, FRANC3D, and FASTRAN II codes operate efficiently on engineering workstations, and FASTRAN II also operates on personal computers.

Experimental Verification of Crack Initiation Methodology

The small-crack effect and crack-closure analysis model, FASTRAN II, were used to calculate the total fatigue life ($S-N$) behavior of single-edge notched (SENT) specimens under constant amplitude and spectrum loads using an initial defect size based on microstructural data at initiation sites. Predicted results for aluminum alloy 2024-T3 were made using an initial semicircular crack size, 0.00024 in. (6 μm), that had an area equal to the average inclusion-particle sizes that were experimentally observed to initiate cracks.³⁰ Comparisons of experimental and predicted fatigue lives of the SENT specimens under the TWIST,³⁶ FALSTAFF,³⁷ and Gaussian³⁸ load sequences are shown in Fig. 7. The specimens were cycled until a crack, length $2a$, had grown across the full thickness B . The predicted lives are in very good agreement with the test data. The results shown in Fig. 7 are for laboratory specimens where all small cracks initiated from microstructural defects at the notch as a result of high local stresses.

Scale 2: Fatigue Crack Growth

The second analysis scale and corresponding computational capability represent the fracture mechanics of fatigue cracks that extend through the thickness of a skin or stiffener, and are no longer three dimensional in their crack-growth behavior.

Crack-Closure Concept for Fatigue Crack Growth

As discussed in the previous section, the plasticity-induced crack-closure model has been shown to be quite accurate in predicting the fatigue crack growth in aluminum alloys for a number of basic crack configurations for both constant amplitude and spectrum loads. The closure model is very accurate for a full range of R ratios and spike overload conditions, provided the crack-growth rate data are correlated with the effective stress-intensity-factor range.

Computational Methodology for Fatigue Crack Growth in Riveted Structure

Two-dimensional analyses are typically quite adequate for predicting crack growth. However, accurate modeling of struc-

tural details is required to provide high-fidelity results for the local stresses in a structure so that the fracture mechanics calculations will be accurate. The FRANC2D finite element analysis code has been developed for the analysis of two-dimensional planar structures,³⁹ and the STAGS (structural analysis of general shells) nonlinear shell analysis code has been developed for general shell structures.⁴⁰ The FRANC2D code, developed by Cornell University, is a user-friendly engineering analysis code with pre- and postprocessing capabilities especially developed for fracture mechanics problems. The code operates on UNIX-based engineering workstations with X-Windows graphics, and is interactive and menu-driven. A unique capability of the code is the ability to predict non-self-similar crack-growth behavior. An automatic adaptive remeshing capability allows an engineer to obtain a history of the stress-intensity factors for any number of cracks in the structure and for any arbitrary crack-growth trajectory. The STAGS finite element code, developed by Lockheed Palo Alto Research Laboratory, provides the capability to model any general shell structure, and has both geometric and material nonlinear analysis capabilities. STAGS is particularly well suited for analyzing shells that have structural features such as frames, stiffeners, and cutouts. The code uses the Riks arc-length projection method and computes large displacements and rotations at the element level. The code has been developed specifically for nonlinear stability and strength analyses. Both FRANC2D and STAGS can calculate the history of the stress-intensity factors for a growing crack that are compatible with FASTRAN II, so that fatigue crack-growth analyses may be performed. Other crack-growth models may also be used. STAGS and FRANC2D operate on engineering workstations and mainframe computers.

The advanced durability and damage-tolerance analysis capabilities developed in the NASA Airframe Structural Integrity Program will also be implemented in the NASGRO (formerly named NASA/FLAGRO) analysis code.⁴¹ NASGRO is a general-purpose damage tolerance analysis code being developed by NASA Johnson Space Flight Center. The code is based on fracture mechanics principles and may be used to compute stress-intensity factors, fatigue crack growth, critical crack sizes, and the limit of safe life. An extensive library of stress-intensity factors may be used with NASGRO, or solutions may be obtained from a boundary element analysis capability using the FADD analysis code.⁴² NASGRO also has an extensive material property library that includes most aluminum alloys, titanium alloys, and steels commonly used in the aerospace industry. Fatigue crack growth may be computed from a crack-closure mechanics model or from one of several empirical models commonly used by industry. NASGRO is used extensively throughout the aerospace industry. FADD, developed at the University of Texas, uses the distributed dislocation method to compute stress-intensity factors. This approach combines a highly accurate stress-intensity-factor analysis with the modeling simplicity of the boundary element analysis method. FADD is also available in a stand-alone version, and is currently being tested by industry at beta-site locations. NASGRO operates on engineering workstations and personal computers. FADD is also available as a stand-alone code and operates on personal computers and workstations.

Experimental Verification of Fatigue Crack-Growth Methodology

In a recent paper,⁴³ the fatigue crack initiation part of the analysis methodology required to predict the onset of WFD was demonstrated.⁴³ Detailed stress analyses of riveted lap-splice joints were conducted using two- and three-dimensional, elastic and elastic-plastic, finite element analyses. Using the local stresses and stress-intensity factors calculated from these analyses, as well as small-crack theory, fatigue analyses were conducted on a series of riveted lap-splice joints. Fatigue lives of uniaxially loaded flat panels with a single riveted lap-splice joint, shown schematically in Fig. 8, were compared with re-

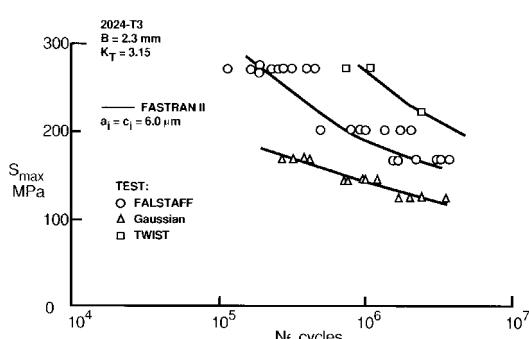


Fig. 7 Comparison of test and predicted $S-N$ behavior.

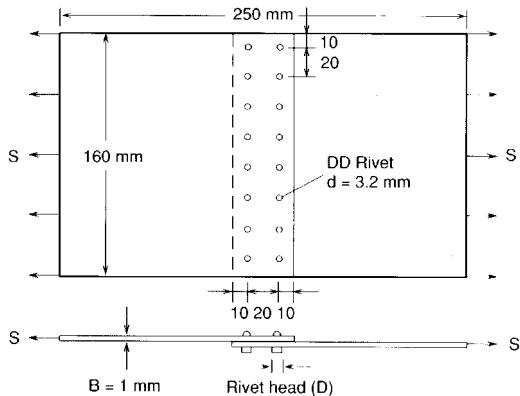


Fig. 8 Test panel with riveted lap-splice joint.

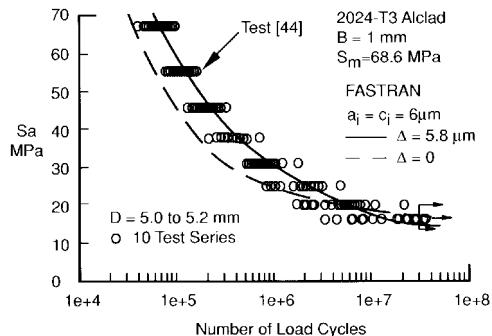


Fig. 9 Comparison of analytical predictions of total fatigue life to experimental results for the riveted test panel.

sults obtained from a comprehensive test program conducted by the National Aerospace Laboratory (NLR) of The Netherlands.⁴⁴ Fatigue lives were calculated from a fracture-mechanics approach, using stress-intensity factors and crack-opening stresses calculated from the FASTRAN code for small cracks subjected to rivet loads, bypass loads, and local bending. The effects of hole preparation were accounted for by the selection of an equivalent initial flaw size. The effects of hole filling were accounted for by the selection of an effective level of interference to compensate for riveting interference, clamp-up, and frictional and residual stress effects. Plasticity effects were only accounted for in the calculation of crack-opening stresses. Linear elastic stress-intensity factors were calculated even when plastic yielding was present at the rivet hole.

A comparison between measured and calculated fatigue lives for the NLR lap joints with a rivet-head diameter of 5.1 mm is shown in Fig. 9. The dashed curve is the predicted fatigue lives based on no interference ($\Delta = 0$), with an initial flaw size of 6 μm . (The 6- μm initial flaw size was found from previous studies of fatigue of open-hole 2024 aluminum alloy specimens.^{30,32}) The interference level was selected by trial and error to fit the experimental test data for each data set. The interference level was 5.8 μm for the 5.1-mm rivet-head diameter. The calculated fatigue lives agreed well with the mean of the test data on the lap joints over a wide range of applied stress levels. If a larger flaw size had been selected, then the interference level needed to fit these test data would also have been larger because the no-interference life calculations would have been shorter than the present calculated lives. Although the level of interference is very low in comparison to quoted values in the literature, the rivet can only exert an elastic spring-back on the rivet hole. Depending upon the level of radial pressure exerted by the rivet (yield stress to several times the yield stress), some estimates of the elastic spring-back ranged from 4 to 16 μm . Further study is needed to substantiate whether small-crack theory and fracture mechanics analyses can predict the onset of WFD in riveted lap-splice joints.

Scale 3: Residual Strength

The third analysis scale and corresponding computational capability represent structures with long cracks that change the internal structural load distribution, exhibit behavior strongly affected by structural details, and affect the residual strength of the structure. In addition, the fracture mechanics of ductile materials such as 2024-T3 aluminum alloy often requires an elastic-plastic stress analysis capability that predicts stable tearing and fracture. Furthermore, nonlinear geometric effects, such as crack bulging in shell structures, also significantly affect residual strength predictions. All of these complexities are present in a fuselage shell structure and must be represented in a residual-strength analysis of the fuselage.

The structural analysis computer codes under development in the NASA Airframe Structural Integrity Program are being integrated into an analytical methodology for predicting the residual strength of a fuselage structure with one or more cracks. The analytical prediction of the residual strength of a complex built-up shell structure, such as a fuselage, requires the integration of a ductile fracture criterion, a fracture mechanics analysis, and a detailed stress analysis of the structure. The CTOA criterion has been experimentally verified to be a valid fracture criterion for mode I stress states in thin and moderately thick (1.27 cm thick or less) aluminum alloys. The CTOA criterion has been demonstrated to be valid for predicting the linkup of a long lead crack with small fatigue cracks ahead of the advancing lead crack. This fracture criterion has been implemented into the STAGS geometric and material nonlinear finite element-based shell analysis code to provide an integrated structural-integrity analysis methodology. The capability to model a growing crack that may extend in a non-self-similar direction has been added to the STAGS code along with an automated mesh refinement and adaptive remeshing procedure. The topological description of the growing crack is provided by the FRANC3D fracture mechanics code. The geometric nonlinear behavior of a stiffened fuselage shell is currently under study for internal pressure loads combined with fuselage body loads that produce tension, compression, and shear loads in the shell.

CTOA Fracture Criterion for Residual Strength

The critical CTOA, or, equivalently, the crack-tip opening displacement (CTOD) fracture criterion is a local approach to characterizing fracture. In contrast, the J-integral or J R-curve criterion is based on global deformations, and has been found to be specimen- and crack-size-dependent for structures with large amounts of stable tearing. The constant CTOA (or CTOD) criterion has been used to predict the variations in J R-curves⁴⁵ as a result of differences in crack sizes and specimen types. Therefore, a local crack-tip displacement is a more fundamental fracture parameter than the J-integral representation for local strain-controlled fracture processes, such as stable tearing and void coalescence. A comparison of the CTOA and the K R-curve, J-integral, and T*-integral may be found in Ref. 46.

Simple plastic-zone models that are based on linear-elastic stress-intensity factors can be adjusted to fit experimental data, and then used to predict crack linkup for relatively simple structural geometries. Although these methods predict the correct trends in crack linkup behavior, they may be difficult to apply to analyses of complex structural details that are characteristic of a fuselage structure. The CTOA criterion can be effectively implemented into a finite element analysis code provided that the code has elastic-plastic deformation and crack-growth simulation capabilities. These capabilities exist in the STAGS geometric and material nonlinear shell analysis code, but analyses of large-scale problems must currently be conducted on a high-performance mainframe computer.

After thorough experimental verification of the residual strength analysis methodology, it is anticipated that the methodology can be simplified by taking advantage of appropriate

engineering approximations. For example, the J R-curve variations for tension- and bend-type fracture specimens were predicted with a constant CTOA fracture criterion.⁴⁵ This analysis demonstrated that neither plane-stress nor plane-strain behavior was adequate in predicting load vs crack extension or load vs displacement from tension- and bend-type fracture specimens. A more detailed three-dimensional finite element stress analysis verified that a constant CTOA value could be used to accurately predict load vs crack extension.⁴⁷ The three-dimensional elastic-plastic stress analysis properly accounted for the effect of thickness constraint on the plastic stress field. The concept of the plane-strain core was subsequently developed, and then used to accurately predict the fracture behavior of both the tension and bend specimens using the same constant CTOA value that was used in the three-dimensional fracture simulations. The plane-strain core approach exploits simple two-dimensional finite element modeling capabilities by imbedding a row of elements with plane-strain behavior in a surrounding mesh of elements with plane-stress behavior. Currently, the plane-strain core size is determined by fitting the analysis to experimental data.

An extensive test program has been conducted to interrogate experimentally the characteristics of the CTOA criterion and to establish its validity as a fracture criterion for thin-sheet 2024-T3 aluminum alloy.⁴⁸ A schematic of the four basic flat-panel geometries used to verify the elastic-plastic finite element code and the CTOA criterion for mode I fracture is shown in Fig. 10. The blunt-notch panel was used to verify the finite element analysis code that computed plastic deformation fields and large displacements. Measurements of far-field displacements and local displacements inside the open holes at the ends of the crack were accurately predicted by the finite element analysis for large-scale plastic deformations.⁴⁸ The center-crack and three-hole-crack panels were used to measure the load (or far-field applied stress) as a function of crack extension and the CTOA during stable tearing. Because the tests were conducted at a specified controlled displacement rate, crack extension was measured well beyond the maximum load observed during the test. Stable tearing was quite extensive in the three-hole-crack specimen because the crack driving force is reduced as the crack approaches the two large open holes in a manner that is similar to the behavior of cracks in stiffened panels. A high-resolution, long-focal-length microscope was used to record the stable-tearing results. The microscope image was videotaped, digitized, and recorded in a computer file. The tearing event was then analyzed on a frame-by-frame basis, and the critical opening angle was measured throughout the fracture event. A typical CTOA measurement is shown in Fig. 11. As can be seen in Fig. 11, the opening angle is relatively insensitive to the length over which the angle is measured. The results of a three-hole-crack panel test are given in Fig. 12. After an initial transition region, the CTOA is constant throughout the stable-tearing process. The

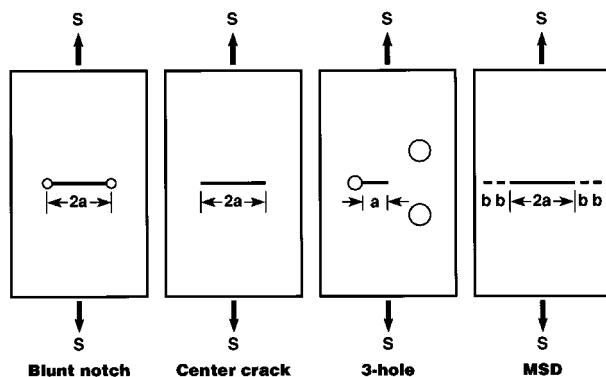


Fig. 10 Experimental verification of the CTOA fracture criterion.

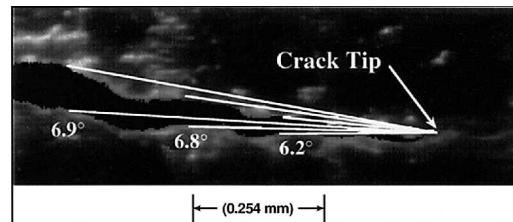


Fig. 11 CTOA measurements.

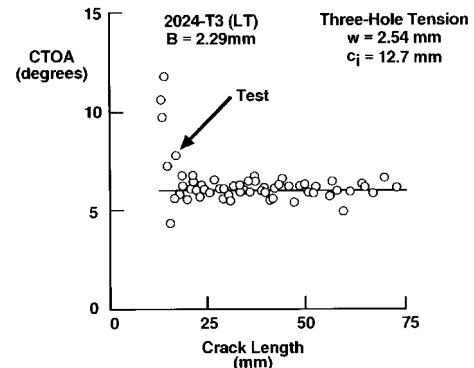


Fig. 12 Experimental measurements of CTOA.

initial transition region is caused by a three-dimensional effect that occurs as the crack tunnels and transitions from flat-to slant-crack growth. More than 63 mm (2.5 in.) of stable tearing was recorded, and the CTOA values were nearly constant. Measurements such as these were also made for center-crack and three-hole-crack panels of various widths, crack lengths, and sheet thicknesses ranging from 1.0 mm (0.04 in.) to 2.3 mm (0.09 in.). Also, measurements of the CTOA were obtained for compact tension specimens. In all cases, the measured CTOA was approximately 6.0 deg for cracks oriented in the *LT* direction of the sheet and 5.1 deg for cracks oriented in the *TL* direction, where *L* designates the principal rolling direction of the sheet and *T* designates the direction transverse to the principal rolling direction. A complete description of these test results is given in Ref. 48.

A series of fracture tests on aluminum panels was conducted by NIST to characterize the fracture behavior and linkup of multiple cracks in very wide panels.¹⁷ Ten flat-panel test specimens 3.988 m (157.0 in.) long, 2.286 m (90.0 in.) wide, and 1.026 mm (0.040 in.) thick were fabricated from single sheets of bare 2024-T3 aluminum alloy. Three center-crack panels with a single long center crack, and seven panels with a long crack and small MSD cracks ahead of the single long crack, were tested to failure. Saw cuts were used to simulate fatigue cracks. Specially designed grips and antibuckling guides were used to conduct the tests in the 1780-kN capacity universal testing machine at NIST. (One test, MSD test 6, was conducted without antibuckling guides, the results of which will be discussed subsequently.) The load and displacement histories were recorded for each test, and the fracture events were recorded on film, videotape, computer, magnetic tape, and occasionally optical microscopy. The first three tests were used to measure basic material fracture properties such as the R-curve and critical CTOA. The other six tests, with antibuckling guides, were linkup and fracture tests of panels with various MSD crack configurations.

The CTOA fracture criterion was used to analytically predict the fracture of the wide panels with MSD cracks. The center-crack panels with the single long crack were used to determine the value of the CTOA to be used in the MSD analyses. To match the load, displacement, and crack extension data recorded during the stable tearing and fracture of the single-crack panels, a critical CTOA value of 3.4 deg had to be used.

In addition, an initial crack opening displacement of 0.0086 in. (0.218 mm) had to be used in the analysis to properly simulate the initial stable tearing from the saw cuts. Using these values of CTOA and initial displacement, the applied load to crack linkup and final fracture were calculated for each MSD test prior to conducting the test. The crack configurations, experimental test loads at panel fracture, and the analytical predictions using the CTOA criterion are given in Table 1. As can be seen, the predictions are within $\pm 6\%$ for all tests but one, which was different by 11%. It is interesting to note that MSD test 10 is a repeat of MSD test 7, with the identical crack configuration. This test is the only MSD test with multiple experimental failure loads. Note that the experimental failure loads varied by about 10%. Therefore, the analytical predictions are viewed to be within the experimental accuracy of the test data.

The CTOA experimentally measured during the fracture tests was consistently about 5.5 deg. This value is significantly higher than the value of 3.4 deg required to analytically simulate the single-crack fracture test behavior. This difference is believed to be attributed to the ineffectiveness of the antibuckling guide to prevent out-of-plane displacements during the fracture tests. Visual observations made during the tests suggested that the panels did buckle even with the antibuckling guides. An additional test, MSD test 6, was conducted without antibuckling guides to determine quantitatively the effect of panel buckling on the fracture load. The test resulted in about a 10% lower failure load than for the panel with the ineffective antibuckling plates. Subsequent fracture analyses conducted by NASA, using the STAGS code and the CTOA fracture criterion, have confirmed that the effects of flat-panel buckling can result in the magnitude of the discrepancy between the measured CTOA and the value required in the analysis to predict the fracture test results.⁴⁹ This study demonstrated that the fracture behavior of a cracked panel that was simultaneously tearing and buckling could be predicted quite well, by using a constant CTOA value determined from a test of a flat panel that was restrained from buckling. Comparisons were made between measured and predicted load vs crack extension and load vs in-plane and out-of-plane displacements. This result has also been confirmed for tests and analyses of flat panels with various MSD crack configurations.

The experimental and analytical results presented herein verify the CTOA fracture criterion for predicting the residual strength of flat panels with cracks undergoing mode 1 fracture behavior. Further testing is required to verify the criterion for predicting the residual strength of complex stiffened shell structures. The CTOA criterion must be extended to mixed-mode loading conditions. Also, numerical procedures for crack extension under mixed-mode loading conditions must be implemented into an elastic-plastic shell analysis code. And finally, the ability to predict crack trajectories accurately and to model curved crack growth must be developed. The next section describes the stiffened shell structural analysis method-

ology being developed for analyzing a fuselage structure and for predicting its residual strength accurately.

Computational Methodology for Residual Strength of Fuselage Structure

A unique capability has been developed that integrates the fracture topology modeling capabilities of FRANC3D with the general shell analysis capabilities of STAGS into an integrated FRANC3D-STAGS analysis procedure.⁵⁰ The automatic adaptive remeshing capability of FRANC3D and the geometric nonlinear stress-analysis capability of STAGS provide the analysis basis required to predict the crack growth, crack turning, and crack arrest behavior exhibited by pressurized shell structures in damage-tolerance tests. The integrated FRANC3D-STAGS analysis procedure currently operates on high-level workstations or on mainframe computers. This capability is described in greater detail in the following text.

The STAGS nonlinear finite element analysis code has been modified to include the capability of conducting crack-growth and residual strength analyses for stiffened fuselage shell structures subjected to combined internal pressure and mechanical loads. STAGS was originally developed to predict the strength, stability, and nonlinear response of nonaxisymmetric or general shells, and includes analyses for both geometric and material nonlinear behavior. The nonlinear solution algorithm used in STAGS is based on Newton's method, and includes both the modified and full versions of that method. Large rotations are represented by a corotational algorithm at the element level, and the Riks arc-length projection method is used to integrate past limit points. The finite element library includes nonlinear beam, plate, and shell elements. Complex stiffened shell structures can be modeled to include as many finite elements as required to represent accurately the response of each structural member in the stiffened shell of interest. The computational efficiency of the code allows nonlinear analyses of models with over 100,000 degrees of freedom to be conducted in a reasonable amount of computer time. Both self-similar and non-self-similar crack-growth prediction capabilities have been added to STAGS for predicting crack growth in a shell that is in a nonlinear equilibrium state. The crack-growth analysis used in FRANC3D-STAGS is based on a virtual crack extension analysis that calculates the strain energy release rate for nonlinear shells with mixed-mode crack growth, including shell wall bending. A load relaxation capability is used to represent the local load redistribution that occurs as a crack grows in the shell, and Newton's method is used to maintain nonlinear equilibrium as the crack propagates. Nonlinear adaptive mesh refinement is used to determine the necessary finite element model changes as the crack propagates.

The general strategy for developing the nonlinear structural analysis methodology for predicting residual strength of stiffened shells with cracks is shown in Fig. 13. Large-scale global models of a stiffened fuselage shell of interest are developed

Table 1 Comparison of measured and predicted failure loads of NIST MSD fracture tests

Panel MSD test	Number of sawcuts	Test load, kips	Predicted load, kips	Percent error
1	1	77.0	76.8 ^a	-0.3
2	1	96.3	96.0	-0.3
3	1	64.9	65.6	+1
4	7	69.1	67.0	-3
5	7	91.2	80.9	-11
7	11	48.2	49.8	+3
8	21	47.5	50.6	+6
9	21	79.2	74.8	-6
10 ^b	11	52.2	49.8	-5

^aFitted to the test data, CTOA = 3.4 deg.

^bPanel MSD test 10 is a repeat of panel MSD test 7.

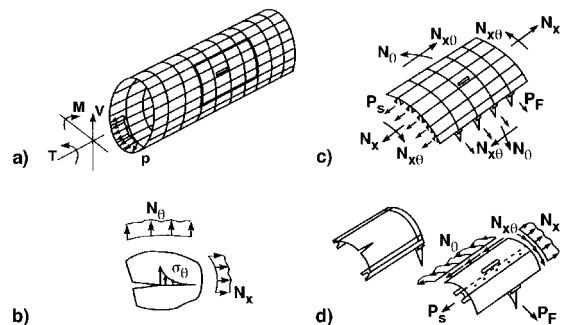


Fig. 13 Hierarchical nonlinear stiffened shell models: a) global shell, b) local detailed stresses, c) stiffened panel, and d) local panel details.

and nonlinear analyses are conducted to determine the internal load distribution and general response of the shell (Fig. 13a). A hierarchical modeling approach is used to provide more highly refined local models, which are developed based on the global model results. The local models provide the higher-fidelity solutions that are necessary to predict stress and displacement gradients near the crack discontinuity in the shell (Fig. 13c). Several local models are generated as required, and are analyzed to provide the detailed stress and deflection results necessary to predict crack growth and residual strength for any structural detail feature, such as the longitudinal lap splice shown in Fig. 13d.

An example⁵¹ of the hierarchical modeling strategy for nonlinear stiffened shell analysis using STAGS is shown in Fig. 14. The nonlinear hoop stress and radial deflection results for the global shell model of a frame and stringer-stiffened aluminum shell are shown on the left of Fig. 14. The shell has a longitudinal crack at the top of the fuselage, and is loaded by 55.2 kPa (8 psi) of internal pressure. The longitudinal crack in the skin is next to a stiffener, and the frame at the crack location is also broken. A curved stiffened panel model was developed with five frames and five stringers, to generate the 36-skin-bay local model shown in the upper right of Fig. 14. This model provides more detailed stress- and deflection-gradient results near the cracked region. The results shown are for a 0.508-m- (20.0-in.-) long skin crack with the center of the crack at the broken frame. The frames are located at the dark circumferential regions in Fig. 14. The boundary conditions for this local model are based on the results of the global model analysis, and both equilibrium and compatibility with the nonlinear global shell solution are maintained at the panel boundaries. A more refined stiffened panel model was developed with two frames and three stringers, to generate the six-skin-bay local model shown in the lower right of Fig. 14. The hoop stress and radial deflection results shown are for a 1.016-m- (40.0-in.-) long crack that has grown to the frames on either side of the broken frame. The boundary conditions for this more refined local model are based on the results of the 36-skin-bay stiffened panel model, and both equilibrium and compatibility with the nonlinear 36-skin-bay panel solution are maintained at the six-skin-bay panel boundaries. This hierarchical modeling and analysis approach provides the high-fidelity nonlinear stress- and deflection-gradient results needed to represent the shell behavior near the crack to the level of accuracy required to predict crack growth and residual strength accurately.

An example of the fracture mechanics analysis capability using FRANC3D-STAGS is shown in Figs. 15 and 16. Using the initial STAGS finite element model (Fig. 15), stable tearing is simulated for a 6-in.- (15.24-cm-), 8-in.- (20.32-mm-), and 10-in.- (25.4-cm-) long skin crack centered over a broken tear strap. The FRANC3D fracture analysis is performed using the local two-bay by two-bay geometry model, with the edge displacements determined from an intermediate six-bay by six-

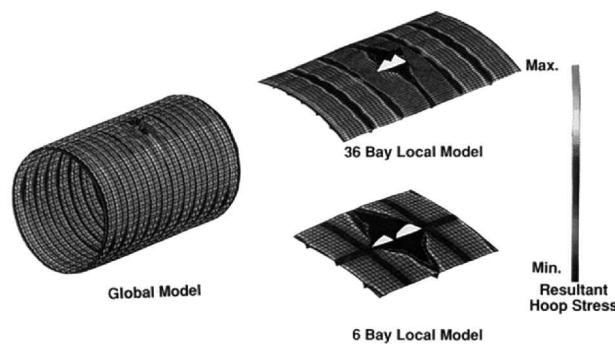


Fig. 14 Stiffened aluminum fuselage shell with 50.8-cm skin crack and broken frame.

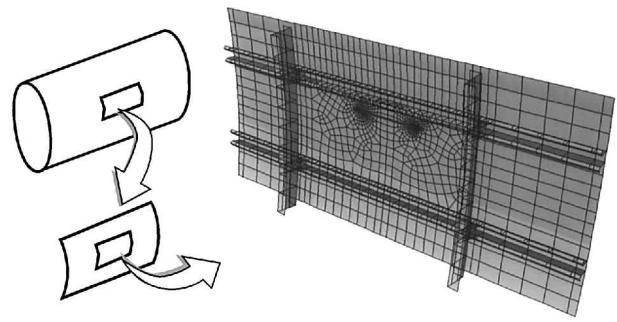


Fig. 15 Curvilinear crack analysis using FRANC3D-STAGS.

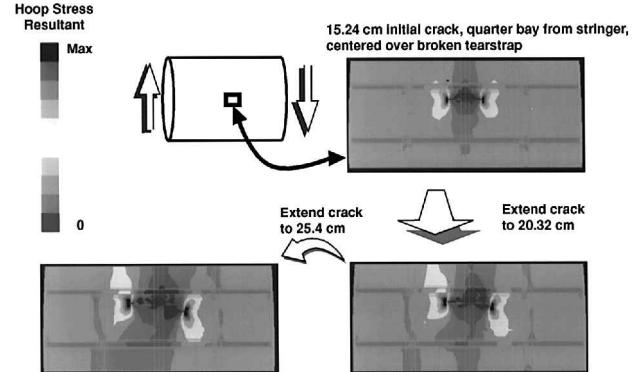


Fig. 16 Curvilinear crack extension: internal pressure plus shear.

bay model. As illustrated schematically in both Figs. 15 and 16, the boundary conditions for the local and intermediate models were determined from the full-scale fuselage cylinder model shown in Fig. 14. The full-scale fuselage model was subjected to an internal pressure of 55.2 kPa (8 psi), plus shear and bending loads simulating downforces acting on the empennage. The stress contour plot in the upper right of Fig. 16 shows the hoop stress resultants for the initial crack length of 6.0 in. (15.24 cm), as modeled by the finite element mesh shown in Fig. 15. Using the FRANC3D adaptive remeshing capability, the crack is extended to 8.0 in. (20.32 cm) and then to 10.0 in. (25.4 cm), as shown in the two lower stress contour plots in Fig. 16. The direction of crack extension was determined by a FRANC3D algorithm using a simple maximum principal stress criterion. Then, the new finite element meshes for the 8- and 10-in. crack geometries were automatically developed by the FRANC3D adaptive remeshing algorithms. The crack-growth increments for this stable tearing simulation were arbitrarily chosen for illustrative purposes only. As is clearly evident from the stress contour plot, the effect of the shear force results in a nonsymmetric and non-self-similar crack extension.

Experimental Verification of Residual Strength Methodology

The methodology to predict analytically the residual strength of a fuselage structure with WFD and discrete source damage will be experimentally verified. NASA will be conducting verification tests of curved stiffened panels simulating sections of the fuselage geometry subjected to internal pressure loads only and subjected to combined loads. Plans call for two tests to be conducted with internal-pressure-only loads. The first test will be a residual strength test on a panel with a midbay skin crack oriented in the longitudinal direction with the damage located away from a splice joint. The second test will be a residual strength test of a panel with simulated WFD in a longitudinal lap-splice joint and with a certification-size lead crack simulating discrete source damage. A third and a fourth verification test will be conducted on panels with the same crack config-

urations as the first two tests, but with combined internal pressure, shear, and bending loads simulating fuselage stresses created by the empennage loads. Two special test facilities at NASA Langley Research Center, a pressure box for internal pressure loads and a D-box for combined internal pressure and mechanical loads, will be used to conduct these tests of the built-up shell structures. In addition to the tests to be conducted at NASA Langley Research Center, test data available from the industry and other government-funded test programs will also be used as benchmarks to verify the accuracy of the analytical prediction methodology. This experimental verification test program will be completed during the next calendar year.

Summary

A comprehensive analytical methodology has been developed for predicting the onset of WFD in fuselage structure. The determination of the aircraft service life that is related to the onset of WFD includes analyses for crack initiation, fatigue crack growth, and residual strength. Therefore, the computational capability required to predict analytically the onset of WFD must be able to represent a wide range of crack sizes, from the material (microscale) level to the global (structural-scale) level. Carefully conducted teardown examinations of aircraft components indicate that WFD behavior can be represented by the following three analysis scales: 1) small three-dimensional cracks at the microscale level, 2) through-the-thickness two-dimensional cracks at the local structural level, and 3) long cracks at the global structural level.

A computational tool box is being developed that includes a number of advanced structural analysis computer codes which, taken together, represent the comprehensive fracture mechanics capability required to predict the onset of WFD. These structural analysis tools have complementary and specialized capabilities ranging from a nonlinear finite element-based stress-analysis code for two- and three-dimensional built-up structures with cracks, to a fatigue and fracture analysis code that uses stress-intensity factors and material-property data found in look-up tables or from equations. The development of these advanced structural analysis methodologies has been guided by the physical evidence of the fatigue process assembled from detailed teardown examinations of actual aircraft structures. In addition, critical experiments are being conducted to verify the predictive capability of these codes, and to provide the basis for any further methodology refinements that may be required. These experiments are essential for analytical methods development and verification, but represent only a first step in the technology-validation and industry-acceptance processes. Each industry user of this advanced methodology must conduct an assessment of the technology, conduct an independent verification, and determine the appropriate integration of the new structural analysis methodologies into their existing in-house practices. NASA has established cooperative programs with U.S. aircraft manufacturers to facilitate the comprehensive transfer of this advanced technology to industry.

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